

Activity 3—Tops, bottoms and middles

ABOUT THIS ACTIVITY

This activity introduces the ideas of central tendency, range and spread. Students calculate the mean, median and mode for ungrouped data. They organise this data into groups and construct a grouped frequency table and histogram of the raw data. They do this to compare the wage structures of two companies where there are differences in variance and spread. Specific Outcomes and Assessment Criteria addressed are: SO2 – AC1, 2, 3, 4, 5.

MANAGING THIS ACTIVITY

Introduce this activity by asking students if they know anything about averages. They will probably remember that to calculate an 'average', we add all items together and divide by the number of items. For example, the average of 20, 35 and 17 is 24 ($20 + 35 + 17 = 72$ and there are three items, so $72 \div 3 = 24$).

This kind of average is called the 'mean' and although it is a very useful measure of central tendency for a set of data, it is also limited. For example, a shoe salesman is not really interested in a mean foot size, but needs to know the most common foot size so that he knows what shoes to order. This is called the 'mode' – the item that occurs most frequently. Employees who want to lobby for higher pay are also not really interested in the mean salary paid by the firm. They are more interested in the 'middle' of the hierarchy of salaries. A very high salary paid to one person can skew the mean, but if the figures are arranged in ascending (or descending) order, and the middle one is selected, the employees have a more useful measure of central tendency. This is called the 'median'. The 'range' of data measures the distance between the extremes of the data - the largest item minus the smallest item.

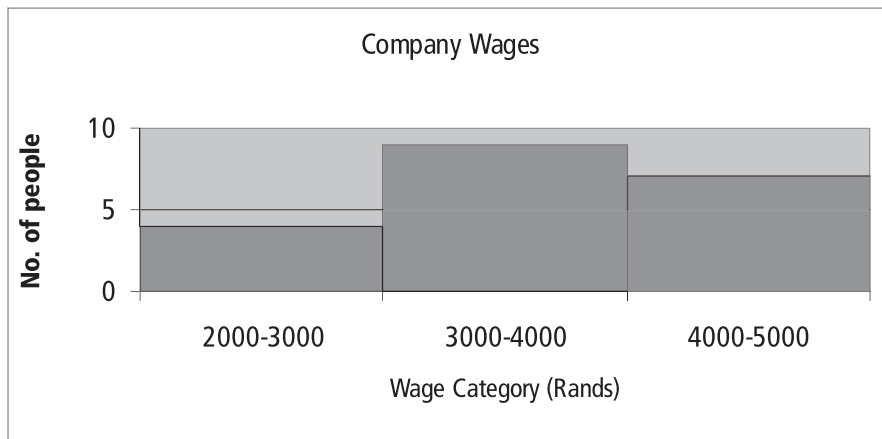
Students should all receive a copy of Worksheet 3.

- 3.1 The highest wage is R4 560, read from the table of raw data.
- 3.2 The lowest wage is R2 370.
- 3.3 The range is found by subtracting the lowest from the highest, so in this case the range is $R4\ 560 - R2\ 370 = R2\ 190$.
- 3.4 The wage that occurs most often is R4 000, read from the raw data.
- 3.5 Count 20 given data items, so there are 20 wages being considered.
- 3.6 Add all the wages to get a total of R71 433. There are 20 items, so the mean is $R71433 \div 20 = R3\ 571,65$.
- 3.7 Arrange the wages in ascending order. We find that there are two items in the middle (3 560 and 3 568), so we add them and halve the result. The mode is therefore $(R3560 + R3\ 568) \div 2 = R3\ 564$
- 3.8 R2 000 – R2 999,99; R3 000 – R3 999,99; R4 000 – R4 999,99 would be suitable groups. Alternatively: R2 000 to <R3 000; R3 000 to <R4 000; R4 000 to <R5 000.

3.9

Wage Category (Rands)	Tally	Frequency
2 000 – 2 999,99		4
3 000 – 3 999,99	###	9
4 000 – 4 999,99	###	7
TOTAL		20

3.10



3.11 4 people out of the 20 are in the lowest category, which is 20%. To calculate percentage multiply the fraction by a hundred: $4 \div 20 \times 100 = 20\%$. In the highest category there are $7 \div 20 \times 100 = 35\%$.

3.12 Almost all the workers earn a low wage of R2 500, but there are two wages above R10 000, which seems unfair. Of course we are not told what the different jobs are, so there may be some justification for the different wages, but the differences are very high.

3.13 Adding all the raw scores, we reach R71 433. We divide by 20 and reach the same mean of R3 571.65. So yes, the statement is true. Highlight for students the interesting fact that very different data can have the same mean.

3.14 Mean – R3 571.65; median – R2 500; mode – R2 500; range is R11 333 - R2 500 = R8 833.

3.15 This is 17 people out of the 20, which is 85% ($17 \div 20 \times 100 = 85\%$).

3.16 The highest wage in Company A is R4 560 and it is $R4 560 - R3 571,65 = R988,35$ away from the mean, but even further from the mean is the lowest wage which is $R3571,65 - R2 370 = R1 201,65$ away from the mean.

3.17 $R11 333 - R3 571,65 = R7 761,35$.

3.18

	Company A (Rands)	Company B (Rands)
Mean	3 571,65	3 571,65
Median	3 564	2 500
Mode	4 000	2 500
Range	2 190	8 833
Furthest from mean	1 021,65	7 761,35
% below the mean	60%	85%

3.19 Each of the companies is concerned with 20 wages and each has an average wage of R3 571,65. However, there are large differences in the range of wages and the individual amounts. This is a good opportunity to introduce the ideas of 'range' and 'spread' informally to students. Company A has a range of only R2 190, while Company B has a large range of R8 833. The most frequently earned salary in Company A, R4 000, is much higher than the most frequently earned salary in Company B, R2 500. Company A has most of its salaries fairly close to the mean, while Company B's salaries are much further from the mean (we say that there is a large spread). A high 85% of Company B's employees earn below the mean, while a lower 60% of Company A employees earn below the mean. The median salary of Company B, R2 500, is quite a bit lower than the median of Company A which is R3564. Employees of Company B, therefore, have a significantly poorer set of wages overall.

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Company A wants to analyse and reorganise its wage structure. The figures below are a sample of the wages measured in rands.

2 800 3 450 4 000 3 550 4 000
 3 560 2 900 4 500 4 100 3 000
 3 568 4 000 3 520 4 560 4 325
 2 370 3 570 3 400 3 700 2 560

- 3.1 What is the highest wage earned in the sample above?
- 3.2 What is the lowest wage?
- 3.3 What is the range?
- 3.4 Which wage occurs most often? What do we call this?
- 3.5 How many people’s wages are being considered?
- 3.6 Calculate the mean wage.
- 3.7 Write the wages in ascending order to help you find the median wage.
- 3.8 The company would like to have three ‘categories’ that all wages would fit into. Suggest three categories that would suit the company’s requirements.
- 3.9 Complete the following frequency table for the raw data given above, using the groups that you suggested:

Wage Category (Rands)	Tally	Frequency
TOTAL		

- 3.10 Draw a histogram showing this data. (A histogram is a bar graph that shows frequency on the vertical axis. The bars are joined as the groups of rands on the horizontal axis do not have any break between them).
- 3.11 What percentage of workers earn in the lowest category? What percentage earn in the highest category?
- 3.12 Company B is in the middle of a wage dispute and the employees recorded the wages in rands as follows:

2 500 2 500 2 500 2 500 2 500
 3 000 2 500 2 500 2 500 2 500
 4 100 2 500 2 500 2 500 2 500
 2 500 2 500 2 500 13 000 11 333

Suggest reasons why there may be a wage dispute at this company.
- 3.13 Management argues that the average wage in Company B is exactly the same as the average wage of Company A. Is this correct?

- 3.14 Give the mean, median, mode and range for Company B.
- 3.15 What percentage of people fall into the R2 000 – R3 000 category?
- 3.16 In Company A, which wage is furthest away from the mean and by how much?
- 3.17 In Company B, which wage is furthest away from the mean and by how much?
- 3.18 Fill in the following table:

	Company A (Rands)	Company B (Rands)
Mean		
Median		
Mode		
Range		
Furthest from the mean		
% of wages below the mean		

- 3.19 Using the ideas and words from the previous questions, describe the differences and similarities between these two sets of wages.